A Frequency-Dependent Finite-Difference Time-Domain Formulation for General Dispersive Media

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***Abstract*— A weakness of the finite-difference time-domain (FDTD) method is that dispersion of the dielectric properties of the scattering/absorbing body is often ignored and frequency-independent properties are generally taken. While this is not a disadvantage for CW or narrow-band irradiation, the results thus obtained may be highly erroneous for short pulses where ultrawide bandwidths are involved. In some recent publications, procedures based on a convolution integral describing $\boldsymbol{D}(t)$ in terms of $\boldsymbol{E}(t)$ are given for media for which the complex permittivity $\epsilon^*(\omega)$ may be described by a single-order Debye relaxation equation or a modified version thereof. Procedures are, however, needed for general dispersive media for which $\epsilon^*(\omega)$ and $\mu^*(\omega)$ may be expressible in terms of rational functions, or for human tissues where multiterm Debye relaxation equations must generally be used. We describe a new differential equation approach, which can be used for general dispersive media. In this method $\boldsymbol{D}(t)$ is expressed in terms of $\boldsymbol{E}(t)$ by means of a differential equation involving $\boldsymbol{D}, \boldsymbol{E}$, and their time derivatives. The method is illustrated by means of one- and three-dimensional examples of media for which $\epsilon^*(\omega)$ is given by a multiterm Debye equation, and for an approximate two-thirds muscle-equivalent model of the human body.**

I. Introduction

THE FINITE-DIFFERENCE TIME-DOMAIN (FDTD) method is being increasingly used for numerical calculations of electromagnetic scattering and absorption. Our interest in the FDTD method has been to apply it to bioelectromagnetic problems, both from the point of view of safety and for medical applications such as hyperthermia. For these applications, the FDTD method has been found to be extremely versatile [1]–[7] and has been used to calculate mass-normalized rates of absorption of electromagnetic energy (specific absorption rates or SARs in W/kg) for spatially uniform or nonuniform incident fields (far-field or near-field) that may be sinusoidally varying (CW) or transient, such as those for an electromagnetic pulse (EMP).

A weakness of the FDTD algorithm is that the dispersion of the dielectric properties of the scattering/absorbing body is mostly ignored and frequency-independent properties are generally taken. While this is not a disadvantage for CW or narrow-band illumination, the results thus obtained may be highly erroneous for short pulses where wide bandwidths are likely to be involved. In our previous work with irradiation of the human body to vertically polarized EMP [6], we had also neglected the known dispersive properties of the biological tissues and had assumed dielectric constants and conductivities for the various tissues at a midband frequency of 40 MHz. Since the induced currents for the various parts of the body were found to have spectral characteristics from sub-MHz frequencies to about 80 MHz for the assumed EMPs with time durations on the order of a few hundred nanoseconds, a question obviously arises about the accuracy of the calculated current waveforms. This is particularly troubling due to the fact that even though the conductivity does not vary a great deal, the dielectric constants of the various high-water-content tissues vary by orders of magnitude over the spectral domain of the induced currents [8]. Yet this variation was neglected in this first set of calculations [6].

In some recent publications, procedures are described for one- and two-dimensional problems for media for which the complex permittivity $\epsilon^*(\omega)$ may be described by a single-order Debye relaxation equation [9], [10] or a modified version thereof [11]. During the revision phase of this manuscript, a recent paper by Sullivan has appeared in the literature, which describes the application of this approach to three-dimensional bodies [12]. As presently developed by these authors, the electrical displacement vector $\boldsymbol{D}(t)$ in the time-domain is written in the form of a convolution integral involving $\boldsymbol{E}(t-\tau)$ and $\epsilon(\tau)$ integrated from 0 to t. Evaluation of a convolution integral will, in general, require storing a large number of past-time values of $\boldsymbol{E}$ for each of the cells, with the resulting need for intolerably large computer memory. For materials that can be described by a single-order Debye relaxation equation (such as water, human tissues for narrow frequency regions, etc.) or a modified version thereof (such as gaseous plasma), $\epsilon(\tau)$ in the time-domain can be written in terms of an exponential function. This allows us to write the convolution integral in terms of a summation of exponential functions, which can be updated recursively and only one additional number will need to be stored for each electric-field component for each of the cells. Our interest is to develop the $(FD)^2TD$ method for the general dispersive media for which $\epsilon^*(\omega)$ may be expressible in terms of rational functions since even for human tissues, a single-order Debye relaxation equation is often not enough and multiterm Debye relaxation equations must therefore be used to describe the dispersion characteristics [13].

In this paper we describe a differential equation approach which should be easy to implement for general dispersive media for which permittivity and/or permeability can be described

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in the frequency domain by means of rational functions. We illustrate the use of this approach by one-dimensional and three-dimensional examples of media for which the complex permittivities are given by a multiterm Debye equation and for an approximate two-thirds muscle-equivalent model of the human body.

II. The Differential-Equation-Based (FD)^{2}TD Method

The time-dependent Maxwell's curl equations used for the FDTD method are:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \tag{2}$$

where the displacement vector $\boldsymbol{D}$ is related to the electric field $\boldsymbol{E}$ through the complex permittivity $\epsilon^*(\omega)$ of the local tissue by the following equation:

$$\boldsymbol{D} = \epsilon^*(\omega)\boldsymbol{E} \tag{3}$$

It should be noted that in writing the Maxwell's equations in the above form, both the conduction and displacement currents are combined in $\partial \boldsymbol{D}/\partial t$ or in defining the complex permittivity $\epsilon^*(\omega)$. In fact, for high-water-content biological tissues, the conduction current is the larger of the two for frequencies of tens of MHz or lower, which is reflected in the imaginary part of ϵ^* being larger than its real part [14]. Since (1) and (2) are to be solved iteratively in the time-domain, it would be desirable to also express (3) in the time-domain.

We have developed a differential equation approach which should be easy to implement for general dispersive media for which permittivity and/or permeability can be described in the frequency-domain by means of rational functions. Because of our interest in bioelectromagnetic problems, we illustrate this procedure by taking an example where $\epsilon^*(\omega)$ can be described by a two-relaxation Debye equation:

$$\epsilon^*(\omega) = \epsilon_o\left[\epsilon_\infty + \frac{\epsilon_{s1} - \epsilon_\infty}{1 + j\omega\tau_1} + \frac{\epsilon_{s2} - \epsilon_\infty}{1 + j\omega\tau_2}\right] \tag{4}$$

From (3) we can write $\boldsymbol{D}(\omega)$

$$\begin{aligned}\boldsymbol{D}(\omega) &= \epsilon^*(\omega)\boldsymbol{E}(\omega) \\ &= \epsilon_o \frac{\epsilon_s + j\omega(\epsilon_{s1}\tau_2 + \epsilon_{s2}\tau_1) - \omega^2\tau_1\tau_2\epsilon_\infty}{1 + j\omega(\tau_1 + \tau_2) - \omega^2\tau_1\tau_2}\boldsymbol{E}(\omega)\end{aligned} \tag{5}$$

where zero (static) frequency dielectric constant ϵ_s is given by

$$\epsilon_s = \epsilon_{s1} + \epsilon_{s2} - \epsilon_\infty. \tag{6}$$

Since (5) is a frequency-domain description of $\boldsymbol{D}$ obtained for a single-frequency sinusoidal variation of fields, we can write it for an arbitrary time variation in terms of the following differential equation:

$$\begin{aligned}\tau_1\tau_2\frac{\partial^2 \boldsymbol{D}}{\partial t^2} + (\tau_1 + \tau_2)\frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{D} \\ = \epsilon_o\left[\epsilon_s \boldsymbol{E}(t) + [\epsilon_{s1}\tau_2 + \epsilon_{s2}\tau_1]\frac{\partial \boldsymbol{E}}{\partial t}\right. \\ \left. + \epsilon_\infty\tau_1\tau_2\frac{\partial^2 \boldsymbol{E}}{\partial t^2}\right]\end{aligned} \tag{7}$$

As can be recognized, (7) is a modified version of the polarization equation in the relaxation theory of dielectrics. For the (FD)^{2}TD method, we need to solve (1) and (2) subject to (7). Similar to references 1, 2, 5, and 15, these equations can be written in the difference form (illustrated for the z-components only for each of the (1), (2), and (7)) shown at the bottom of the page where $\tilde{\boldsymbol{H}} = \sqrt{\mu_o/\epsilon_o}\boldsymbol{H}$

$$a_o = \frac{\epsilon_o\epsilon_s}{2}, \alpha_1 = \epsilon_o(\epsilon_{s1}\tau_2 + \epsilon_{s2}\tau_1), \alpha_2 = \epsilon_o\epsilon_\infty\tau_1\tau_2$$
$$\beta_o = \frac{1}{2}, \beta_1 = (\tau_1 + \tau_2), \beta_2 = \tau_1\tau_2.$$

$$\tilde{H}_z^{n+1/2}(i+1/2, j+1/2, k) = \tilde{H}_z^{n-1/2}(i+1/2, j+1/2), k) + \frac{c\delta t}{\delta}\left[\begin{array}{l} E_x^n(i+1/2, j+1, k) - E_x^n(i+1/2, j, k) \\ +E_y^n(i, j+1/2, k) - E_y^n(i+1, j+1/2, k)\end{array}\right] \tag{8}$$

$$D_z^{n+1}(i, j, k+1/2) = D_z^n(i, j, k+1/2) + \sqrt{\frac{\epsilon_o}{\mu}}\frac{\delta t}{\delta} \cdot \left[\begin{array}{l} \tilde{H}_y^{n+1/2}(i+1/2, j, k+1/2) - \tilde{H}_y^{n+1/2}(i-1/2, j, k+1/2) \\ +\tilde{H}_x^{n+1/2}(i, j-1/2, k+1/2) - \tilde{H}_x^{n+1/2}(i, j+1/2, k+1/2)\end{array}\right] \tag{9}$$

$$\begin{aligned}\alpha_o(E_z^{n+1} + E_z^n) + \frac{\alpha_1}{\delta t}(E_z^{n+1} - E_z^n) + \frac{\alpha_2}{(\delta t)^2}(E_z^{n+1} - 2E_z^n + E_z^{n-1}) \\ = \beta_o(D_z^{n+1} + D_z^n) + \frac{\beta_1}{\delta t}(D_z^{n+1} - D_z^n) + \frac{\beta_2}{(\delta t)^2}(D_z^{n+1} - 2D_z + D_z^{n-1})\end{aligned} \tag{10}$$

In (8)–(10), the subscript n denotes the time $n\delta t$ in terms of the incremental time or time step δt. The time step δt is determined by the cell size δ (assumed equal in x, y, and z directions. To satisfy the stability condition, the following condition is generally taken: $\delta t = \delta/2c_{\max}$ where $c_{\max}$ is the maximum velocity of the electromagnetic waves encountered anywhere in the interaction space. For the present calculations, we have taken $c_{\max} = c$ corresponding to velocity of EM waves in air.

Upon rearranging terms, we can rewrite (10) as follows

$$\left[\alpha_o + \frac{\alpha_1}{\delta t} + \frac{\alpha_2}{(\delta t)^2}\right] E_z^{n+1} = \left(\frac{2\alpha_2}{(\delta t)^2} + \frac{\alpha_1}{\delta t} - \alpha_o\right) E_z^n - \frac{\alpha_2}{(\delta t)^2} E_z^{n-1} + \left[\beta_o + \frac{\beta_1}{\delta t} + \frac{\beta_2}{(\delta t)^2}\right] D_z^{n+1} + \left[\beta_o - \frac{\beta_1}{\delta t} - \frac{2\beta_2}{(\delta t)^2}\right] D_z^n + \frac{\beta_2}{(\delta t)^2} D_z^{n-1}. \quad (11)$$

As in the conventional (frequency-independent) FDTD method, the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are calculated in a time-stepping manner for a lattice of cubic cells similar to the method originally proposed by Yee [16]. In our formulation we use the values of $\boldsymbol{E}$ to calculate $\tilde{\boldsymbol{H}}$ from (8) and similar x- and y-component equations of (1) in the difference form; use $\tilde{\boldsymbol{H}}$ to calculate $\boldsymbol{D}$ from (9) and other component equations of (2); and use $\boldsymbol{D}$ to calculate $\boldsymbol{E}$ from (11) for the following time step, after which the process is repeated for the next time step and so on.

In a heterogeneous model, $\alpha_o, \alpha_1, \alpha_2, \beta_1$, and β_2 will be different for different cells depending on the tissues in that individual cell. The corresponding coefficients of (11) would, therefore, need to be stored for such models. Such heterogeneous models have not, however, been considered to date. We are examining the possibility of fitting the frequency variations of the measured complex permittivities (ϵ^*) for various tissues to the same relaxation time constants, τ_1 and τ_2 for all of the tissues so that the ϵ^* of the various tissue compositions that typically exist for the various cells may be combined. With this approach, only three quantities, viz, α_o, α_1, and α_2 will need to be stored for the individual cells.

III. One-Dimensional Test Cases

1. Air-Water Interface

For water, the complex permittivity $\epsilon^*(\omega)$ can be described by a single-order Debye relaxation equation

$$\epsilon^*(\omega) = \epsilon_o\left[\epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + j\omega\tau_0}\right] \quad (12)$$

where ϵ_o is the permittivity of free space ($= 8.85 \times 10^{-12}$ F/m), ϵ_s and ϵ_∞ are the dielectric constants at zero (static) and

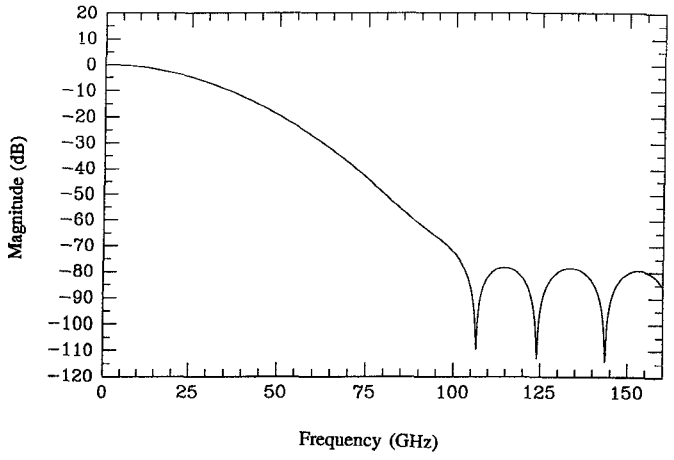


Fig. 1. Frequency spectrum of the truncated Gaussian pulse used for the 1-D air-water interface problem. $E(t) = 1000e^{-(t-t_o)^2/T^2}$, where $t_o = 400\delta t, T = 152\delta t$.

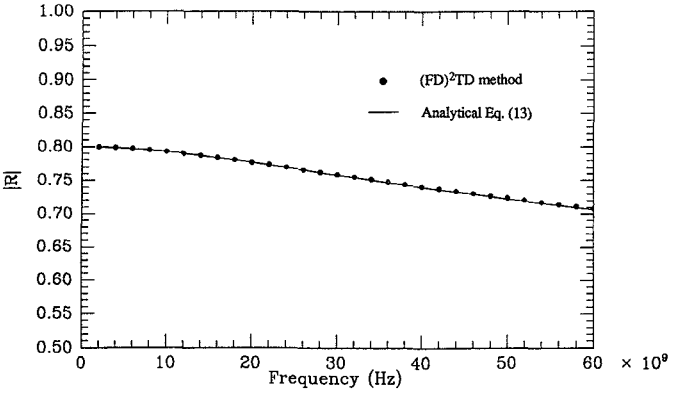


Fig. 2. Comparison of the reflection coefficient for normal incidence at an air-water interface computed using the (FD)^{2}TD algorithm for broadband pulse irradiation and from the analytical equation (13).

"infinite" frequencies, and τ_o is the "relaxation-time" constant. We can adapt the formulation of (7) to this case by putting $\epsilon_{s1} = \epsilon_s, \epsilon_{s2} = \epsilon_\infty, \tau_1 = \tau_o$ and $\tau_2 = 0$, in which case, (7) becomes a first-order differential equation.

Similar to [9], we have assumed $\epsilon_s = 81, \epsilon_\infty = 1.8$, and $\tau_o = 9.4 \times 10^{-12}$ [17]. We also divided the one-dimensional problem space into 1000 cells, 499 of which were used to model air and 501 to model water. As in [9], we also took a cell size δ of 37.5 μm and the time step $\delta t = \delta/2c = 0.0625$ ps, where c is the velocity of EM waves in air.

The assumed incident pulse was of the form $E(t) = 1000e^{-(t-t_o)^2/T^2}$ where $t_o = 400\delta t$ and $T = 152\delta t$. The frequency spectrum for this Gaussian pulse is shown in Fig. 1, where the lobing structure is due to the truncation of the pulse.

We have used the differential-equation-based (FD)^{2}TD method to calculate the reflection coefficient as function of frequency for the air-water interface. Shown in Fig. 2 is a comparison of the reflection coefficient at various frequencies obtained from the (FD)^{2}TD method and the exact values calculated using the following analytical equation:

$$|R(\omega)| = \left| \frac{\sqrt{\epsilon_o} - \sqrt{\epsilon^*(\omega)}}{\sqrt{\epsilon_o} + \sqrt{\epsilon^*(\omega)}} \right|. \tag{13}$$

An excellent agreement is found between the numerical results using the (FD) ^{2}TD procedure and the analytical values for the reflection coefficient at various frequencies.

2. Air-Muscle Interface

To test the applicability of the differential-equation-based (FD)^{2}TD method for dispersive media of more complex frequency variability, we applied it to another one-dimensional problem for the air-2/3 muscle interface. Since the average permittivity of the human body is close to that for 2/3 muscle, we decided to take this permittivity instead of that for the muscle.

A five-relaxation Debye equation given in the following has previously been used to fit to the experimental data for the muscle [13]

$$\epsilon^* = \epsilon_o \left[4.3 + \frac{8 \times 10^5}{1 + jf/69} + \frac{81\,900}{1 + jf/(43 \times 10^3)} + \frac{11\,900}{1 + jf/(0.67 \times 10^6)} + \frac{32}{1 + jf/(230 \times 10^6)} + \frac{45.8}{1 + jf/(20 \times 10^9)} \right] \tag{14}$$

where f is the frequency in Hz. Since a five-relaxation Debye equation such as (14) would result in a fifth-order differential equation for $\boldsymbol{D}(t)$ in terms of $\boldsymbol{E}(t)$ and its derivatives, which would require a larger storage of the various quantities for each of the cells, we have attempted to fit a two-relaxation Debye equation similar to (4) to the experimental data for the muscle. We have been able to obtain a relatively decent fit to the experimental data for the frequency band 20 MHz to 20 GHz by using the following equation:

$$\epsilon^*(\omega) = \epsilon_o \left[19 + \frac{10\,000}{1 + jf(0.71 \times 10^{-6})} + \frac{42}{1 + jf(0.75 \times 10^{-10})} \right] \tag{15}$$

Shown in Figs. 3(a) and (b) as solid curves are the variations given by (15) for both the relative permittivity ϵ_r (real part of ϵ^*/ϵ_o) and the conductivity $\sigma(= \omega)$ imaginary part of ϵ^*). Also shown for comparison are the average values of the experimental data summarized in reference 13 and the variations given by the five-relaxation Debye (14). For the frequency band 20 MHz to 20 GHz, the variations given by (15) are in reasonable agreement with the experimental data for the muscle. A major advantage of using the simpler (15) is that it is possible to use a second-order differential equation for time-domain representation of $\boldsymbol{D}(t)$ rather than a fifth-order differential equation that would be needed had we used the full-blown (14) from [13].

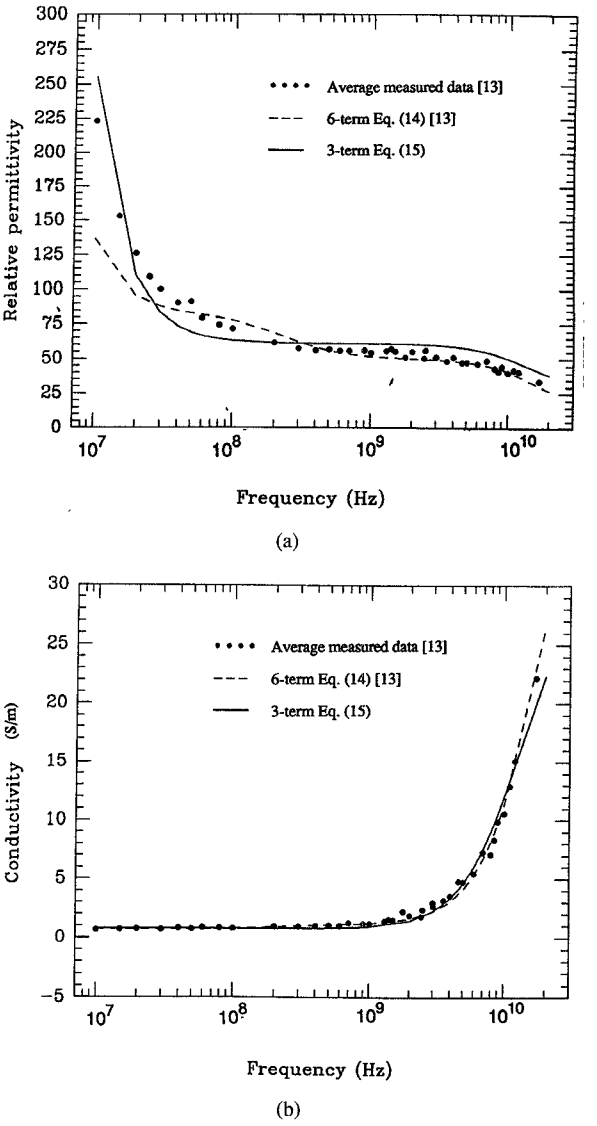


Fig. 3. Frequency variation of the electrical properties of muscle. (a) relative permittivity ϵ_r. (b) conductivity σ.

Similar to the previous test case of air-water interface, we divided the problem space into 1000 cells of which 499 were in air and the remaining 501 were in the 2/3 muscle-equivalent material. The cell size taken for the calculations was 0.0127 cm ($< \lambda_\epsilon/10$ for $f \leq 40$ GHz) and $\delta t = \delta/2c = 0.21$ ps. The incident Gaussian pulse had a peak amplitude of 1000 V/m and was of the form $E(t) = 1000e^{-(t-t_o)^2/T^2}$ where $t_o = 3T$ and $T = 71\delta t$. The frequency spectrum of the assumed pulse is shown in Fig. 4.

In Fig. 5, we compare the reflection coefficient for the air-2/3 muscle interface obtained for the various frequencies using the (FD)^{2}TD method with that obtained using the analytical (13). Also shown for comparison are the values that would be obtained for the various frequencies had the conventional FDTD method been used and properties assumed corresponding to a midband frequency of, say, 10 GHz. While the (FD)^{2}TD method using a single run with, say, a

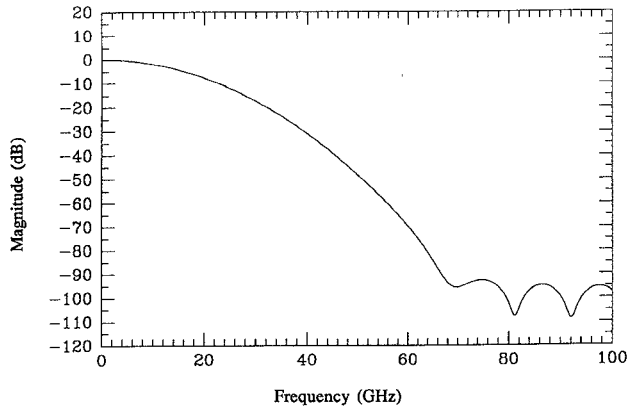


Fig. 4. Frequency spectrum of the truncated Gaussian pulse used for the 1-D air-muscle interface problem. $E(t) = 1000e^{-(t-t_o)^2/T^2}$, where $t_o = 3T, T = 71\delta t$.

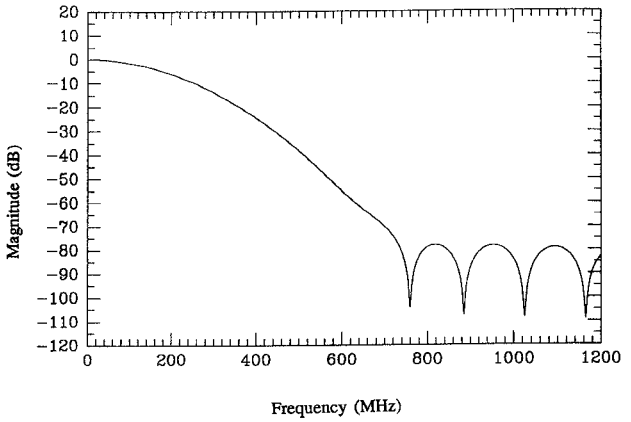


Fig. 6. Frequency spectrum of the truncated Gaussian pulse used for the 3-D problem using a sphere of 2/3 muscle-equivalent material. $E(t) = 1000e^{-(t-t_o)^2/T^2}$, where $t_o = 210\delta t, T = 80\delta t$.

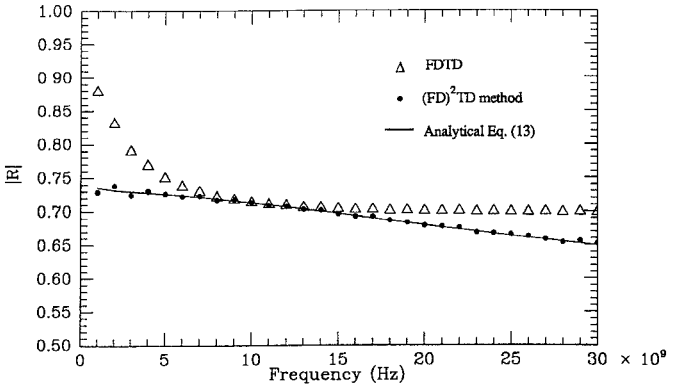


Fig. 5. Comparison of the reflection coefficient for normal incidence at the air-2/3 muscle interface computed using the (FD)^{2}TD algorithm with broadband pulse irradiation and for the traditional FDTD method where properties at the midband frequency of 10 GHz are taken.

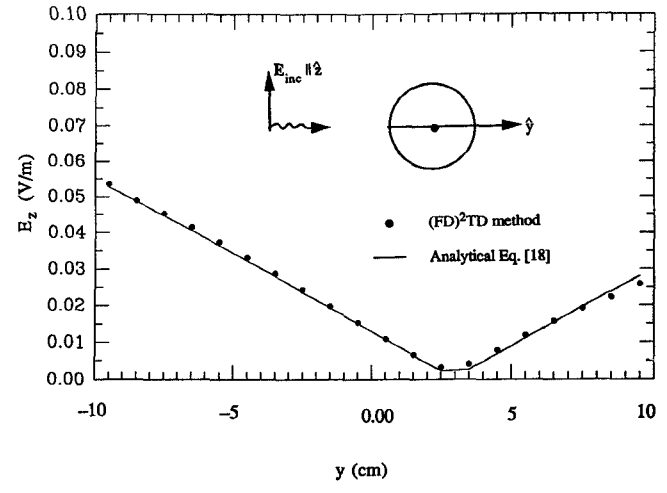
Gaussian pulse, gives excellent agreement with the analytical values at all the frequencies, the conventional FDTD method gives agreement for the frequency band where the dielectric properties are close to the values that are assumed for the calculations. As seen in Fig. 5, for dispersive media such as the biological tissues, severe errors of calculated results occur for both lower- and higher-frequency regions.

IV. Three-Dimensional Test Case: Sphere of 2/3 Muscle-Equivalent Material

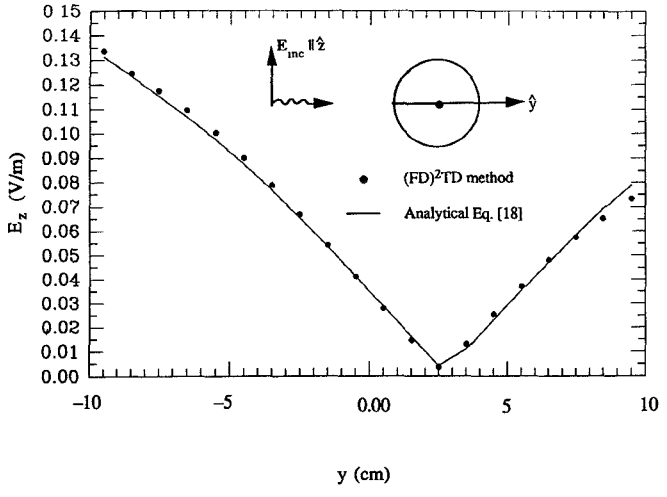
We have used the test case of a 2/3 muscle-equivalent sphere and compared the results of the internal E-field distributions at various frequencies with the analytical Mie series solutions that can be obtained at the corresponding frequencies [18]. For the dimensions of the sphere, we have taken a diameter of 20 cm corresponding to the average dimensions of the human head. We have divided the three-dimensional sphere into cells of size 1 cm and have taken a space of 6 cells from the boundaries of the sphere to the absorbing boundaries on each of the sides in the x-, y-, and z-directions, respectively. The time step δt is taken to be 1/60 ns. Because of the larger cell size of 1 cm ($< \lambda_\epsilon/10$ for $f \leq 350$ MHz), a Gaussian with lower frequency components is taken as the incident pulse. The assumed Gaussian is of the form $1000e^{-(t-t_o)^2/T^2}$ where $t_o = 210\delta t$ and $T = 80\delta t$. The frequency spectrum of the Gaussian pulse is given in Fig. 6. In Figs. 7(a), (b), and (c), we compare the (FD)^{2}TD-calculated variations of E_z along the y-axis at some representative frequencies with those obtained from the Mie series solutions. The agreements of the calculated E-field variations using the (FD)^{2}TD method with the analytical solutions are excellent for frequencies for which the cell size δ is less than $\lambda_\epsilon/10$. Though not shown for lack of space, similar agreements have also been obtained for other field components for various locations of the sphere.

V. Calculations for the Human Model

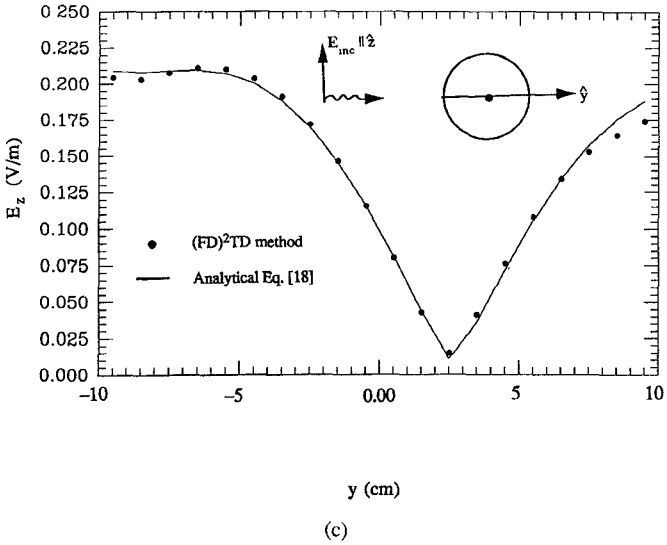
We have used the differential-equation-based (FD)^{2}TD method of calculate the RF currents induced in the model of the human body. For these calculations we have used the anatomically based model of the human body described in our earlier publications [1]–[6], except that uniform 2/3 muscle-equivalent ϵ^* was assumed for each of the cells internal to the body for this first set of calculations. As in the past, volume-averaged properties were used for all of the cells at the model boundary, since these cells are only partially composed of 2/3 muscle-equivalent material, while the rest of it is air. If the tissue content of a given boundary cell is less than 10% by volume, air was assumed to be the material for such cells. For all of the calculations to date we have used cubical cells of size 2.62 cm. As in [6], the modeled space is divided into $38 \times 26 \times 84 = 82\,992$ cells, of which 5628 cells are either totally or partially within the human body. A spacing of 7–9 cells from the model is taken to the absorbing boundaries for the various sides of the modeled space. For the calculations shown in Fig. 8(a) and (b), we have assumed isolated (ungrounded) condition of the human model and a flat-top impulse of radiation of width $5\delta t$ in the time-domain, where $\delta t = 0.04367$ ns is the time step used for the calculations. The frequency spectrum of this impulse is fairly



(a)



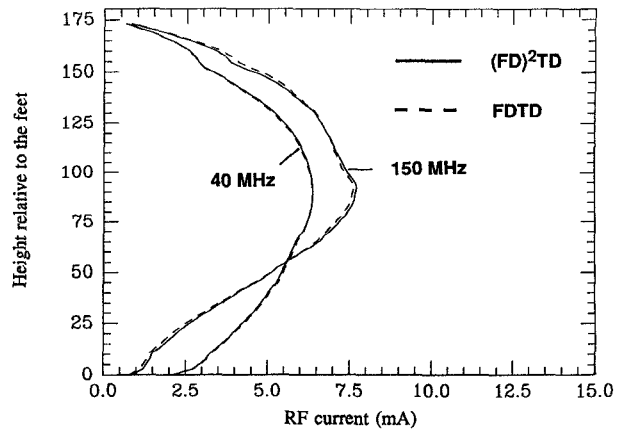
(b)



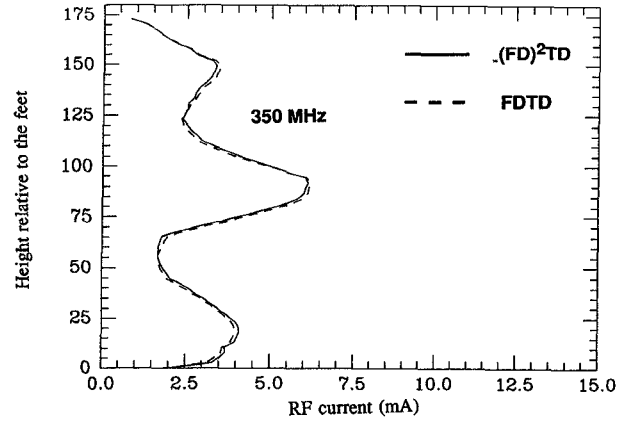
(c)

Fig. 7. Comparison of E_z calculated by the (FD)^{2}TD method with the analytic results using Mie series [18]. Diameter of the 2/3 muscle-equivalent sphere = 20 cm, corresponding to the average dimensions of the human head. (a) $f = 40$ MHz. (b) $f = 100$ MHz. (c) $t = 200$ MHz.

broad with a nearly constant amplitude for frequencies up to 500 MHz, which is in excess of the region of validity of this model with cell size of 2.62 cm.



(a)



(b)

Fig. 8. Induced RF current distributions for a 2/3 muscle-equivalent model of the human body. Model assumed isolated from ground. $E_{\text{inc}} = 1$ V/m for each of the frequencies. (a) 40 MHz, 150 MHz. (b) 350 MHz.

We have used the calculated vertically directed internal D_z fields to calculate local z-directed current densities, including both the conduction and displacement currents, for each of the cells using the relationship $J_z = \partial D_z/\partial t$. The vertically directed currents for any of the layers were then obtained by summing the terms due to the individual cells in a given layer as follows

$$I = \delta^2 \sum_i \frac{\partial D_z}{\partial t} \tag{16}$$

where δ^2 is the cross-sectional area $(2.62 \times 2.62\,\text{cm}^2)$ for each of the cells in the body.

The induced RF current distributions are calculated for some representative frequencies 40, 150, and 350 MHz, and are shown as solid curves in Fig. 8(a) and (b). The results for each of the frequencies are normalized for an incident E field of 1 V/m at the corresponding frequencies. Shown for comparison are the current distributions obtained for the same model using the conventional FDTD procedure. For the latter calculations, a flat-top impulse of irradiation as described above was also used, except that different properties for the model correspond-

ing to 2/3 muscle-equivalent material (from (15)) were used for the three frequencies. Three individual runs naturally had to be made for the FDTD method, since different properties for ϵ^* were involved at the three frequencies. As seen in Fig. 8(a) and (b), excellent agreement is obtained for the $(FD)^2TD$ results with those obtained using the conventional FDTD method, with the added advantage of being able to use a single run rather than multiple runs needed for the FDTD method.

VI. Conclusions

In this paper we have described a differential-equation-based frequency-dependent finite-difference time-domain ($(FD)^2TD$) method, which can be used for general dispersive media for which $\epsilon^*(\omega)$ and $\mu^*(\omega)$ may be expressible in terms of rational functions. We have illustrated the use of this method by one- and three-dimensional examples of media for which $\epsilon^*(\omega)$ is given by a multiterm Debye equation, and for an approximate two-thirds muscle-equivalent model of the human body. Using a single run involving a prescribed incident pulse in the time domain (e.g., a Gaussian pulse or a flat-top impulse), the $(FD)^2TD$ method allows calculations of coupled EM fields, $\boldsymbol{E}, \boldsymbol{H}$, and $\boldsymbol{D}$, and induced currents as a function of time. By taking the Fourier components of the induced fields, one can then obtain the corresponding components at the various frequencies.

For coupling to extremely narrow pulses such as may be encountered for ultra wideband radar, one can also store the impulse response of the scatterer and convolve the fields thus obtained with the prescribed shape of the incident pulse. This procedure has recently been used [19] to calculate induced currents in the 1.31-cm cell size model of the human body (45 024 cells representing the human body) for exposure to electromagnetic pulse (EMP) and has the advantage of not requiring repeated large-memory FDTD runs.

References

[1] D. M. Sullivan, D. T. Borup, and O. P. Gandhi, "Use of the finite-difference time-domain method in calculating EM absorption in human tissues," *IEEE Trans. Biomed. Eng.*, vol. BME-34, pp. 148–157, 1987.

[2] D. M. Sullivan, O. P. Gandhi, and A. Taflove, "Use of the finite-difference time-domain method in calculating EM absorption in man models," *IEEE Trans. Biomed. Eng.*, vol. BME-35, pp. 179–186, 1988.

[3] J. Y. Chen and O. P. Gandhi, "RF currents induced in an anatomically based model of a human for plane-wave exposures 20-100 MHz," *Health Physics*, vol. 57, pp. 89–98, 1989.

[4] ——, "Electromagnetic deposition in an anatomically based model of man for leakage fields of a parallel-plate dielectric heater," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 174–180, 1989.

[5] C. Q. Wang and O. P. Gandhi, "Numerical simulation of annular-phased arrays for anatomically based models using the FDTD method," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 118–126, 1989.

[6] J. Y. Chen and O. P. Gandhi, "Currents induced in an anatomically based model of a human for exposure to vertically polarized EMP," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 31–39, Jan. 1991.

[7] O. P. Gandhi and J. Y. Chen, "Numerical dosimetry at power-line frequencies using anatomically based models," *Bioelectromagnetics*, supplement 1, pp. 43–60, 1992.

[8] M. A. Stuchly and S. S. Stuchly, "Electrical properties of biological substances," in *Biological Effects and Medical Applications of Electromagnetic Energy*, O. P. Gandhi, Ed., Englewood Cliffs, NJ: Prentice-Hall, ch. 5, pp. 75–112, 1990.

[9] R. J. Luebbers, F. P. Hunsberger, K. S. Kunz, R. B. Standler, and M. Schneider, "A frequency-dependent finite-difference time-domain formulation for dispersive materials," *IEEE Transactions on Electromagn. Compat.*, vol. 32, pp. 222–227, Aug. 1990.

[10] M. D. Bui, S. S. Stuchly, and G. I. Costache, "Propagation of transients in dispersive dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1165–1172, July 1991.

[11] R. J. Luebbers, F. P. Hunsberger, and K. S. Kunz, "A frequency-dependent finite difference time-domain formulation for transient propagation in plasma," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 29–34, Jan. 1991.

[12] D. M. Sullivan, "A frequency-dependent FDTD method for biological applications," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 532–539, Mar. 1992.

[13] W. P. Hurt, "Multiterm debye dispersion relations for permittivity of muscle," *IEEE Trans. Biomed. Eng.*, vol. BME-32, pp. 60–63, 1985.

[14] M. A. Stuchly and S. S. Stuchly, "Dielectric properties of biological substances—tabulated," *J. Microwave Power*, vol. 15, pp. 19–26, 1980.

[15] A. Taflove and K. R. Umashankar, "The finite-difference time-domain method for numerical modeling of electromagnetic wave interactions with arbitrary structures," in *Finite-Element and Finite-Difference Methods in Electromagnetic Scattering*, M. A. Morgan, Ed. New York: Elsevier, ch. 8, pp. 287–373, 1990.

[16] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302–307, 1966.

[17] K. S. Cole and R. H. Cole, "Dispersion and absorption in dielectrics," *J. Chem. Phys.*, vol. 9, pp. 341–351, 1941.

[18] J. R. Mautz, "Mie series solution for a sphere," *IEEE Trans. Microwave Theory Techn.*, vol. 26, p. 375, 1978.

[19] J. Y. Chen and O. P. Gandhi, "A simple convolution procedure for calculating currents induced in the human body for exposure to EMP," presented at the *Thirteenth Annual Meeting of the Bioelectromagnetics Society*, Salt Lake City, Utah, June 23–27, 1991.

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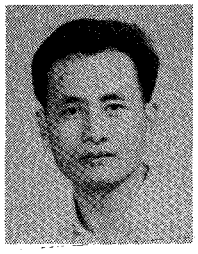

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